APPENDIX - A

CURVES

BUREAU OF DESIGN AND ENVIRONMENT

SURVEY MANUAL

May 2001

APPENDIX - A

CURVES

The radius can be used to define a curve in lieu of the degree of curve definition to describe horizontal curves.

For projects designed or constructed in English units of measure the arc definition of degree of curve is primarily used. The arc definition is defined as the change of direction of the central angle per an arc length of 100 feet. The relationship between the degree of curve and the radius in feet is stated by the following equation:

$$R = \frac{5729.58}{D}$$

Some of the various definitions and relationships of a simple curve are illustrated in Figure A.1, page A-9 and given as follows:

(Δ) = Deflection Angle

Tangent distance (T) = $R(\mathrm{Tan}\ \Delta/2)$

External distance (E) = $R(\mathrm{Sec}\ \Delta/2 - 1) = R(\mathrm{Exsec}\ \Delta/2) = T(\mathrm{Sin}\ \Delta/2) - M$

Middle Ordinate (M) = $R(1 - \mathrm{Cos}\ \Delta/2) = R(\mathrm{Vers}\ \Delta/2) = R - R(\mathrm{Cos}\ \Delta/2)$

Long Chord (L.C.) = $2R(\mathrm{Sin}\ \Delta/2) = 2T(\mathrm{Cos}\ \Delta/2)$

Length of Curve (L) = $\frac{\Delta(100)}{D} = \frac{\Delta 2\pi R}{360^\circ} = \frac{\Delta R}{57.2958}$

The deflection angle (Δ) is measured in decimal degrees. For English units D is measured in decimal degrees and R is in feet.

In addition to simple curves, compound curves, reverse curves and vertical curves are also used in highway work.

Compound curves are a combination of two or more simple curves and their use should be avoided where a simple curve can be used. However, due either to right-of-way problems or topographic considerations, a compound curve may occasionally be necessary. The curve must not be compounded at a ratio greater than 2:1. If the adjacent curves differ by more than 2º-00', a transition curve should be used.

A reverse curve is a combination of two simple curves of opposite curvature with a common tangent. A tangent adequate in length to provide the superelevation transition required by the Design Policies should be provided between the curves. If the reverse curves do not contain any superelevations, a tangent between the curves is not required.

A "Broken-back" curve is a term used to denote two curves in the same direction separated by a short tangent or by a flat curve whose radius is greater than twice the radius of either of the two initial curves. This layout is particularly objectionable on highways and should be avoided by using one simple curve or a compound curve if necessary.

The curves described above are illustrated in Figure A.2, page A-10.

In the past, there was a period when transition or spiral curves were extensively used in connection with pavement widening at curves. As the curves became flatter and the pavement wider, it became unnecessary to widen at curves in this manner. It remains, however, that vehicular paths entering and leaving circular curves follow a spiral curve. For this reason or for reasons of "fitting" an alignment into a problem area, transition curves may occasionally be used.

Vertical curves are used to transfer the smoothly change from one slope to another along an alignment to account for the change in terrain. See Figure A.3, page A-11 for a diagram of a vertical curve.

CALCULATIONS FOR A HORIZONTAL CIRCULAR CURVE

Given:

P.I. Sta. 107+67.90, $\Delta = 11° \; 00' \; 00''$, $D = 2° \; 30' \; 00''$

Calculate the Radius

$R = 5729.58/D$

$R = 5729.58/2° \; 30' \; 00''$

$R = 2291.83'$

Tangent Distance

$T = R(\text{Tan } \Delta/2)$

$T = 2291.83(\text{Tan } 11° \; 00' \; 00''/2)$

$T = 220.68'$

Length of Curve

$L = 100 \; (\Delta/D)$

$L = 100 \; (11° \; 00' \; 00''/ \; 2° \; 30' \; 00'')$

$L = 440.00'$

External Distance

$E = T(\text{Tan } \Delta/4)$

$E = 220.68(\text{Tan } 11° 00' 00''/4)$

$E = 10.60'$

CALCULATE P.C. AND P.T. STATIONS

P.C. Station

P.C. = P.I. Station – Tangent Distance

P.C. = 107+67.90 – 220.68

P.C. = 105+47.22

P.T. Station

P.T. = 105+47.22 + 440.00

P.T. = 109+87.22

CALCULATE DEFLECTION ANGLES

Deflection for 100' of Arc

100' Arc = D/2

100' Arc = 2° 30' 00"/2

100' Arc = 1° 15' 00"

Deflection for 50' of Arc

50' Arc = D/4

50' Arc =2° 30' 00"/4

50' Arc =0° 37' 30"

Deflection for 25' of Arc

25' Arc = D/8

25' Arc =2° 30' 00"/8

25' Arc = 0° 18' 45"

Deflection for 1' of Arc

1' Arc = D/200

1' Arc = 2° 30' 00"/200

1' Arc = 0° 00' 45"

CALCULATE CHORD LENGTHS

Chord Length for 100' of Arc

100' Arc = (2)(R)(Sin of Deflection Angle) Deflection Angle = D/2

100' Arc = (2)(2291.83)(Sin 1° 15' 00")

100' Arc = 99.99'

Chord Length for 50' of Arc

50' Arc = (2)(R)(Sin of Deflection Angle) Deflection Angle = D/2

50' Arc = (2)(291.83)(Sin 0° 37' 30")

50' Arc = 50.00'

CALCULATE THE DEFLECTION FOR THE FIRST STATION FROM THE P.C. OR ANY ODD STATION ALONG THE CURVE

1. Take the distance from the last point with a known deflection to the station you are calculating.
2. Multiply this distance by the deflection of a 1' Arc (D/200), this will give you the deflection between these two points.

   Example: Find the deflection angle for Sta. 108+55.

   (108+55 - 105+47.22) = 307.78'

   307.78'(0° 00' 45") = (3° 50' 50") Note: Use decimal degrees for this calculation.

EXAMPLE OF A FIELD BOOK SETUP FOR A HORIZONTAL CURVE

Deflections For Curve #1

Sta.	Distance	Chord Distance	Deflection Angle	Total Deflection	
105+00					
+47.22	0	0	0	0	P.C.
+50	2.78'	2.78'	0° 02' 05"	0° 02' 05"	
106+00	50.0	50.00	0° 37' 30"	0° 39' 35"	
+50	50.0	50.00	0° 37' 30"	1° 17' 05"	
107+00	50.0	50.00	0° 37' 30"	1° 54' 35"	
+50	50.0	50.00	0° 37' 30"	2° 32' 05"	
108+00	50.0	50.00	0° 37' 30"	3° 09' 35"	
+50	50.0	50.00	0° 37' 30"	3° 47' 05"	
109+00	50.0	50.00	0° 37' 30"	4° 24' 35"	
+50	50.0	50.00	0° 37' 30"	5° 02' 05"	
+87.22	37.22	37.22	0° 27' 55"	5° 30' 00"	P.T.

Calculated by (Initials) (Date)
Checked by (Initials) (Date)

Δ = 11° 00'00" D = 2° 30' 00"
P.C. Marked by P.K. Nail

Deflection Angles
100' of Arc = D/2 = 2° 30' 00"
= 1° 15' 00"
50' of Arc = D/4 = 2° 30' 00"/4
= 0° 37'30"
1' of Arc = D/200 = 2° 30' 00"/200
= 0° 00' 45"

Chord Lengths
100' of Arc = (2)(R)(Sin of Deflection Angle)
= (2)(2291.83)(Sin 1° 15' 00")
= 99.99'
50' of Arc = (2)(R)(Sin of Deflection Angle)
= (2)(2291.83)(Sin 0° 37' 00")
= 50.00'

P.T. (Note: Total Deflection should equal Δ/2)

CALCULATIONS FOR VERTICAL CURVE

Please refer to Figure A.3 on page A-11 of this appendix for a graphic view of the components of a vertical curve. Following are several definitions of the elements of a vertical curve and a set of sample field notes that have been prepared for field use to stake out a vertical curve.

Definitions:
PVC: “Point of vertical curve”. Station on centerline where the vertical curve starts.
PVI: “Point of vertical intersection”. Station at which the two tangent grade lines intersect.

PVT. "Point of vertical tangency". Station on centerline where the vertical curve ends.
LVC. Length of vertical curve.
OFFSET: The vertical distance from the tangent grade line to the vertical curve.
e: a mathematical constant whose value is determined by the grades of the two intersecting tangents and the length of the vertical curve.
e: Grade #2(%) – Grade#1 (%) x (LVC) (Stations)/8 (Constant) = (G2-G1)(LVC/100)/8
CL ELEV: = Tangent Elevation $\pm$ offset.
Highpoint/low point locations.
Distance from PVC = G_1 x (Stations)/G_2 –G_1

Sample set of field notes for a vertical curve.

Station	**(X) Distance**	Tangent **Elevation**	**(Y)offset**	Elevation **on Curve**
45+50		429.34		
46+00	50.00	428.84	0.03	428.87
47+00	150.00	427.84	0.26	428.10
48+00	250.00	426.84	0.71	427.55
49+00	350.00	425.84	1.40	427.24
49+50	400.00	425.34	1.83	427.17
50+00	450.00	424.84	2.31	427.15
50+50	500.00	424.34	2.86	427.20
51+00	550.00	423.84	3.46	427.30
52+00	650.00	422.84	4.83	427.67
52+50	700.00	422.34	5.60	427.94
53+00	650.00	423.44	4.83	428.27
54+00	550.00	425.64	3.46	429.10
55+00	450.00	427.84	2.31	430.15
56+00	350.00	430.04	1.40	431.44
57+00	250.00	432.24	0.71	432.95
58+00	150.00	434.44	0.26	434.70
59+00	50.00	436.64	0.03	436.67
59+50		437.74		437.74

e = $(G_2\text{-}G_1)$LVC/8 = (2.2-(-1.0))14/8 = 3.2(14)/8 = 5.60 Vertical offset at VPI.

Y-Offset @ 46+00 & 59+00 = $(50/700)^2(5.60)$ = 0.03
47+00 & 58+00 = $(150/700)^2(5.60)$ = 0.26
48+00 & 57+00 = $(250/700)^2(5.60)$ = 0.71 Y = e(X/L)2
49+00 & 56+00 = $(350/700)^2(5.60)$ = 1.40
49+50 = $(400/700)^2(5.60)$ = 1.83
50+00 & 55+00 = $(450/700)^2(5.60)$ = 2.31
50+50 = $(500/700)^2(5.60)$ = 2.86
51+00 & 54+00 = $(550/700)^2(5.60)$ = 3.46
52+00 & 53+00 = $(650/700)^2(5.60)$ = 4.83
52+50 = $(700/700)^2(5.60)$ = 5.60

When using this method the elevation difference is calculated from both the tangent gradients at specified distances from the PVC and PVT and then applied to the tangent elevations determined for the two gradients. The value of (e) is the offset at the VPI and each individual station's offset value is determined as a percentage of the external at the VPI. The vertical offsets from a tangent to a parabola are proportional to the squares of the distances from the point of tangency.

STATION AND ELEVATION OF LOW POINT OF VERTICAL CURVE

X (In Station from VPC) = $G_1L/(G_2\text{-}G_1)$ = 1.0(14)/(2.2-(-1.0)) = 1.0(14)/3.2 = 4.375 Sta. or 437.50'.
Station of Low Point = 45+50 + (4+37.5) = 49+87.50
X-Distance = 437.50 Tangent Elev. = 429.34 – 437.50(0.01) = 424.96'
Y-Offset $(437.50/700)^2(5.60)$ = 2.19'
Elevation of Low Point = 424.96 + 2.19 = 427.15'

ALTERNATE METHOD OF CALCULATING CURVE ELEVATIONS

An alternative method of calculating the elevations of a vertical curve is as follows: calculate the value of (a) using the following formula: The tangent elevations are computed using the elevation of the PVC and the slope of the forward tangent. An elevation is computed for each station needed. Then compute the offsets from the forward tangent to the curve. The offset equals **aX^2**. Apply the offset values to the tangent elevation to obtain the curve elevation.

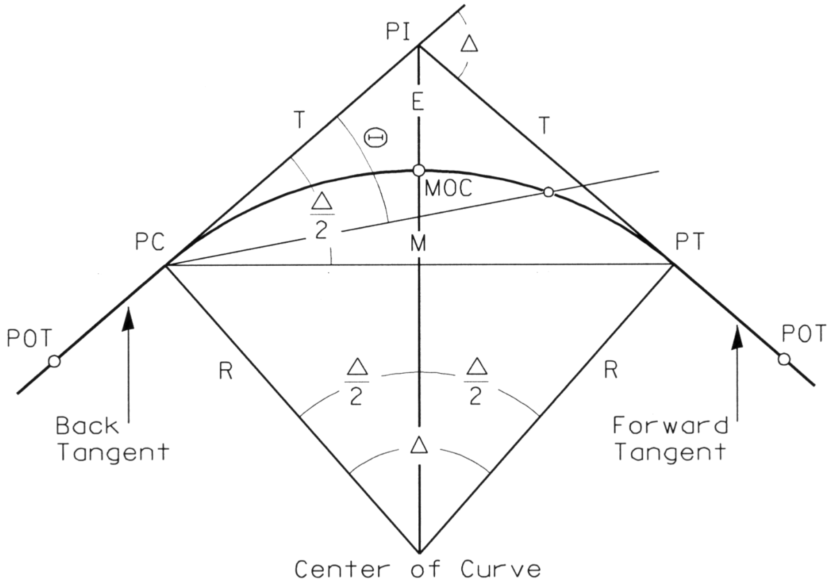
a = $100(G_2\text{-}G_1)/2L$ = 100(2.2-(-1.0))/2*1400 = 0.114
2a = 0.228

Station	**Tangent** **El. =G_1X**	**Offsets from AV=aX^2**	**Curve. Elev.**	**Check 1st Diff.**	**2nd Diff**
45+50	429.34		429.34		
46+00	428.84	$-.114*0.50^2$ =0.03	428.87		
47+00	427.84	$-.114*1.5^2$ =0.256	428.10	-0.77	
48+00	426.84	$-.114*2.5^2$ =0.71	427.55	-0.55	0.22
49+00	425.84	$-.114*3.5^2$ =1.40	427.24	-0.31	0.24
49+50	425.34	$-.114*4.0^2$ =1.83	427.17		
50+00	424.84	$-.114*4.5^2$ =2.31	427.15	-0.09	0.22
50+50	424.34	$-.114*5.0^2$ =2.86	427.20		
51+00	423.84	$-.114*5.5^2$ =3.46	427.30	+0.15	0.24
52+00	422.84	$-.114*6.5^2$ =4.83	427.67	+0.37	0.22
52+50	422.34	$-.114*7.0^2$ =5.60	427.94		
53+00	421.84	$-.114*7.5^2$ =6.43	428.27	+0.60	0.23
54+00	420.84	$-.114*8.5^2$ =8.26	429.10	+0.83	0.23
55+00	419.84	$-.114*9.5^2$ =10.31	430.15	+1.05	0.23
56+00	418.84	$-.114*10.5^2$ =12.60	431.44	+1.29	0.24
57+00	417.84	$-.114*11.5^2$ =15.11	432.95	+1.51	0.22
58+00	416.84	$-.114*12.5^2$ =17.86	434.70	+1.75	0.24
59+00	415.84	$-.114*13.50^2$ =20.83	436.67	+1.97	0.22
59+50	415.34	$-.114*14.00^2$ =22.40	437.74		

One method of checking your calculations is to calculate the first and second differences of the curve elevation between the full stations. The second difference should be the same and is equal to 2a, which is the percent of constant change in slope per station.

If other methods of calculating the station vertical offsets are desired, see a survey textbook for the procedures.

FORMULAS & EXAMPLES - SIMPLE CIRCULAR CURVE



DEFINITIONS Be sure the instrument and carrying case are kept dry. If they become wet, allow them to air dry before closing the carrying case. Extend level rod and let air dry overnight.

Back Tangent	=	Tangent from which the curve starts
Forward Tangent	=	Tangent on which the curve ends
POT	=	"Point on Tangent" - Any point on the tangent portion where the curve starts or ends
PC	=	"Point of Curvature" - Station on centerline where the curve starts
T	=	"Tangent" - The distance on a straight line from the PC to the PI or the PT to the PI
MOC	=	"Mid-Point of Curve"
PT	=	"Point of Tangency" - Station on centerline where the curve ends
L	=	"Length of Curve" - The distance <u>along the curved centerline</u> from the PC to the PT
PI	=	"Point of Intersection" - The point where the back tangent and the forward tangent intersect
R	=	"Radius of the Curve"
E	=	"External Distance" - Distance from the MOC to the PI
M	=	"Middle Ordinate" - Distance from the MOC to the mid-point of the straight line between the PC and the PT (the LC)
LC	=	"Long Chord" - Straight line distance from the PC to the PT
Δ	=	The Central Angle of the Curve - The angle between a radial line from the center of the curve to the PC and a radial line from the center of the curve to the PT; also equals the angle of intersection of the forward tangent with the back tangent

Figure A.1

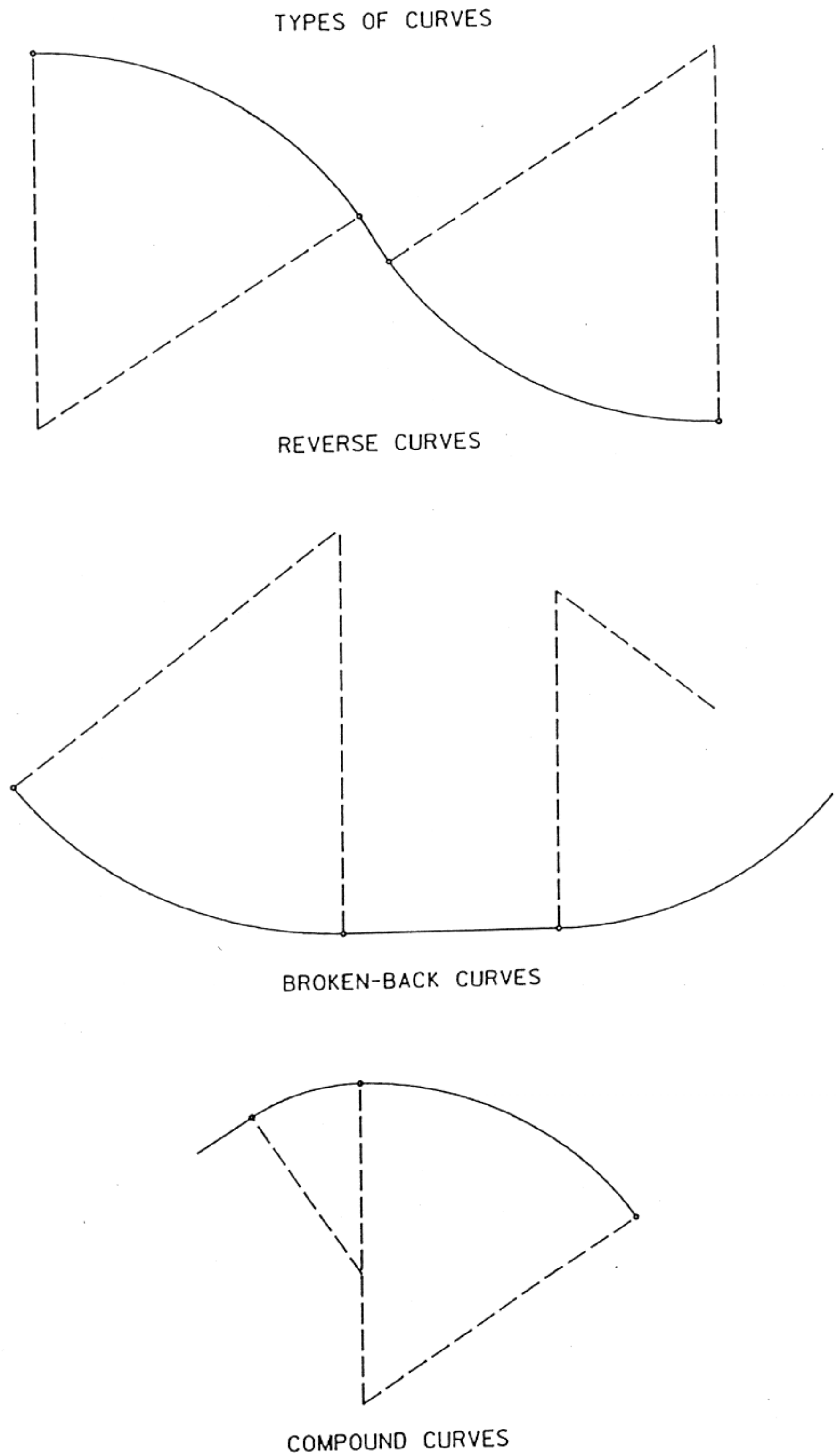
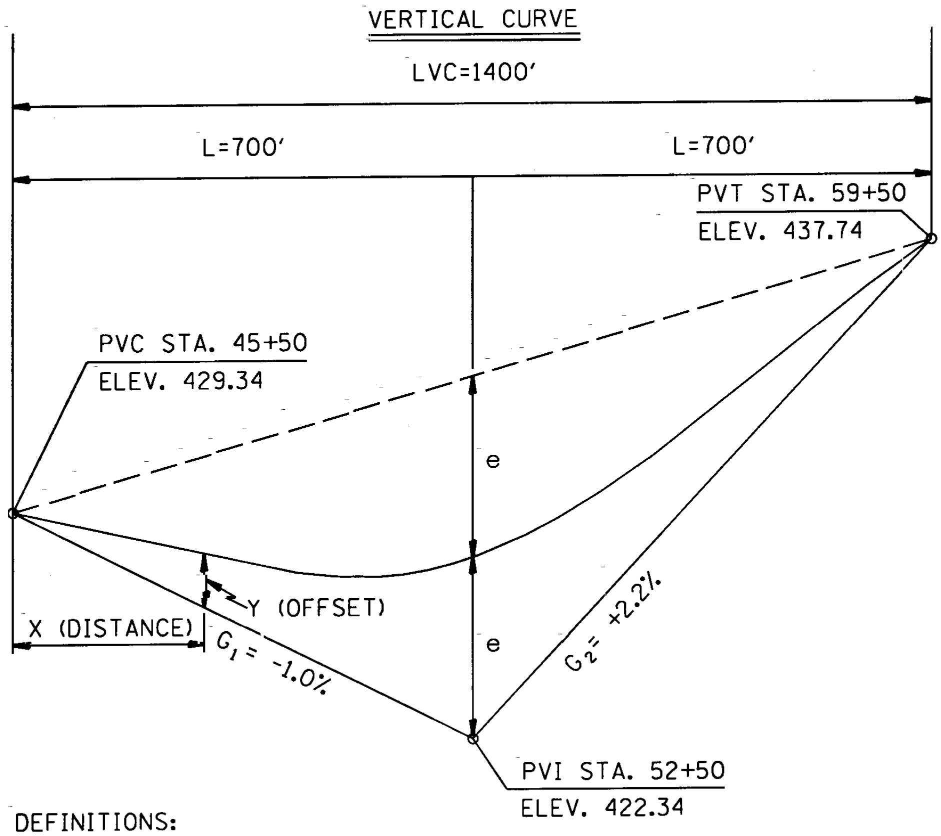

TYPES OF CURVES
REVERSE CURVES
BROKEN-BACK CURVES
COMPOUND CURVES

Figure A.2



DEFINITIONS:

"POINT OF VERTICAL CURVE" - STATION ON CENTERLINE WHERE THE VERTICAL CURVE STARTS

"POINT OF VERTICAL INTERSECTION" - STATION AT WHICH THE TWO TANGENT GRADE LINES INTERSECT

"POINT OF VERTICAL TANGENCY" - STATION ON CENTERLINE WHERE THE VERTICAL CURVE ENDS

= "LENGTH OF VERTICAL CURVE"

OFFSET = THE VERTICAL DISTANCE FROM THE TANGENT GRADE LINE TO THE VERTICAL CURVE

e = A MATHEMATICAL CONSTANT WHOSE VALUE IS DETERMINED BY THE GRADES OF THE TWO INTERSECTING TANGENTS AND THE LENGTH OF THE VERTICAL CURVE

$$e = \frac{\text{GRADE \#2 (\%) - GRADE \#1 (\%) X LVC (STATIONS)}}{\text{8 (CONSTANT)}}$$

℄ ELEVATION = TANGENT ELEV. ± OFFSET

HIGH POINT/LOW POINT LOCATION

DISTANCE FROM PVC = G_1 X LVC (STATIONS)/G_2-G_1

Figure A.3